Economics 352: Intermediate Microeconomics

Notes and Sample Questions
Chapter 15: Game Theory Models of Pricing

The book goes through a lot of this stuff in a more technical sense. I’ll try to be plain and clear about it.

The Basics
1. Players
Players are the parties playing the game. They may be people, teams, firms, nations, animals, bacteria or just about anything else that is capable of executing a strategy.

2. Strategies
Strategies are the actions available to the players.

3. Payoffs
Payoffs are the returns to the players at the end of the game.

4. Equilibrium (or a Nash Equilibrium)
This is a situation in which no one player wishes to change her choice of strategy given what the other player(s) are doing.

For example, everyone driving on the right side of the road is an equilibrium, because anyone who chose to drive on the left side of the road would die almost instantly.

As another example, the oddly named demilitarized zone separating North and South Korea is an equilibrium, because any person who tries to cross it will be killed almost instantly.

Extensive and Normal Forms
A game may be presented in either extensive form, as a game tree, or in normal form, though a payoff matrix.

Before discussing game trees and payoff matrices, it is worth making the distinction between a simultaneous game in which players move simultaneously, or without knowledge of what their opponent has done, and sequential games, in which one player moves and then his opponent observes what his move was and then reacts to it.

A payoff matrix is generally used to illustrate a simultaneous game, whereas game trees may illustrate either type of game.

The example given in the book has the following payoff matrix:
For example, if Player A chooses the strategy L while Player B chooses the strategy S, then the payoff to Player A will be 5 and the payoff to Player B will be 4.

The same game can be illustrated using a game tree, as shown below:

If Player A chooses strategy S and Player B chooses strategy L, for example, then the payoff to Player A will be 6 and the payoff to Player B will be 4.

The dotted ellipse around the two nodes at which Player B makes a decision indicates that when Player B makes his choice, he does not know which strategy Player A has chosen. This is consistent with the game being a simultaneous game.

If we were to use the game tree to illustrate this as a sequential game, in which Player A moves first and Player B then observes and reacts to Player A’s strategy, then the game would be more correctly drawn without the ellipse as:
Games With Dominant Strategies
If one player in a game has a strategy that is always best, regardless of what the other player does, this is called a dominant strategy and you can count on that player making that move.

Consider the example below:

Person A has the dominant strategy “down” and Person B has the dominant strategy “right.” The equilibrium is down,right, despite the fact that the best outcome (in terms of total combined payoff) is up,left.

Consider the example below:
Looking at Player 1’s options, Down is always better than Up, so Player 1 will choose Down. Given that Player 1 will choose Down, Player 2 can choose Left and get a payoff of 8 or can choose Right and get a payoff of 3. She will choose Left, giving us an equilibrium of Down,Left.

### Games Without Dominant Strategies

Now, consider the following game. It has two equilibria: Down,Left and Up,Right.

Which might be chosen is difficult to say if you don’t know more about the game.

This is very much like the “which side of the road do we drive on” game.
Consider the well known "Battle of the Sexes" game. Two players, let's call them Allen and Elizabeth, are trying to decide what to do one evening. Allen wants to go see an exciting hockey game while Elizabeth, for reasons Allen can't understand, wants to go to some art gallery. They argue about it that morning and leave for work without deciding what to do. For reasons too complicated to explain here, they can't communicate during the day, so each is faced with the problem of where to go after work. For reasons too complicated to explain here, each would like to be with the other, though each would also prefer to be at his or her preferred activity. Their payoffs from each choice are described in the following payoff matrix:

If the players in this game move simultaneously, there is no pure strategy equilibrium. That is, the equilibrium involves each player going to one place or the other with some probability.
If the game is played sequentially, however, we will get different equilibria depending one which player moves first. In this case, a player moving first must commit to going to one place and then communicate that commitment to the other player. In this case, Allen would go to one place and then call Elizabeth at work and tell her he was there and couldn't go to the other event. Alternatively, Elizabeth could get to one place first and then call Allen.

Solving by reverse induction (that is, solving the game from the last period back to the first) we see that if Allen moves first, both of them wind up at the hockey game while if Elizabeth moves first they go to the art gallery. Thus, in this game, if one player can credibly and irreversibly commit to moving first, they will benefit from that commitment.

To be clear about reverse induction, in the version of the sequential game where Allen moves first, the analysis begins with Elizabeth’s move. If she knows that Allen has gone to the hockey game, she can go to the hockey game and get a payoff of 2 or go to the art museum and get a payoff of 1, so she will go to the hockey game. Similarly, if she knows that Allen has gone to the art museum, she can go to the art museum and get a payoff of 5 or go to the hockey game and get a payoff of 0, so she will go to the art museum. Now, Allen knows her payoffs and knows that if he goes to the hockey game, Elizabeth will go there as well and he will get a payoff of 5, whereas if he goes to the art museum, she will go there as well and he will get a payoff of 2. As a result, he will go to
the hockey game, where Elizabeth will meet him. He will get a payoff of 5 and she will get a payoff of 2.

To help in solving games through reverse induction, it helps to go through and highlight the strategy that each player will choose from each node (the dots on the game tree), working backward from the last period to the first. For the first version of the battle of the sexes game presented above, this looks like:

Allen now knows that if he chooses the hockey game, Elizabeth will also choose the hockey game and he will get a payoff of 5. On the other hand, if he chooses the art museum, Elizabeth will also choose the art museum and he will get a payoff of 2. Allen will choose the hockey game, because this gives him the larger payoff.

As an example of this, consider the following story from when I was teaching at a remote location near a swamp. As I walked out to my pickup truck one evening after teaching, a voice said, “Give me your keys or I’ll stab you.”

I immediately went into sequential game theory mode. I could hand over the keys and lose my truck, or I could toss the keys into the swamp. The criminal could then choose to either stab me or not, although stabbing me would make her crime even more serious than it already was, which she probably didn’t want. Anyhow, I devised the following game tree:
For me, the best outcome is that I don’t give up the keys and don’t get stabbed. The next best is giving up the keys but not getting stabbed. The next best is not giving up the keys and getting stabbed (I get to keep my truck that way) and the worst outcome is giving up the keys and getting stabbed anyway, which could happen.

For the criminal, the best outcome would be for me to give up the keys and not stabbing me. The second best outcome would be to not get the keys, but also to not commit the very serious crime of stabbing me. The third best outcome would be to stab me (committing a serious crime) but getting my truck. The worst outcome for the thief would be to not get the truck and also to commit the serious crime of stabbing me.

The payoffs represent the ranking described above.

Working backward, what equilibrium do you find?

**Mixed Strategy Equilibria**
A mixed strategy equilibrium exists in a simultaneous game when neither player has a dominant strategy.

Saying that a game has a mixed strategy equilibrium is a fancy way of saying that players should randomly choose their strategies.

A great example of a game with a mixed strategy equilibrium is the rock/scissors/paper game illustrated in the textbook. Any consistent strategy might be discovered by an
opponent, so randomly choosing which strategy to use (rock, paper or scissors) is the best way to go.

Two other games in which neither player has a dominant strategy are the battle of the sexes games presented above and in the book and, also, the game of chicken.

The game of chicken is based on the story of two high school gentlemen who obtain cars and drive them straight at each other on a road. Each has the strategies of “swerve” or “don’t swerve”. If one swerves and the other doesn’t, the one who swerves suffers some disgrace and dishonor. If both swerve then they tie and there is no winner. If neither swerves, then they crash and suffer tremendous injuries. The payoff matrix looks like this:

<table>
<thead>
<tr>
<th></th>
<th>swerve</th>
<th>don't</th>
</tr>
</thead>
<tbody>
<tr>
<td>swerve</td>
<td>0</td>
<td>+10</td>
</tr>
<tr>
<td>0</td>
<td>-10</td>
<td></td>
</tr>
<tr>
<td>Player A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>don't</td>
<td>-10</td>
<td>-100</td>
</tr>
<tr>
<td>+10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In all of these games, no player has a dominant strategy, so there is not what economists call a pure strategy equilibrium, or an equilibrium in which you know for sure what people will do.

Instead, players will randomly choose one strategy or another with different probabilities in what is called a mixed strategy equilibrium.

The key to finding a mixed strategy equilibrium is to calculate your probabilities so that the other player is indifferent between his two choices. In the book’s example 15.2, diagrammed in figure 15.2, B’s payoffs from his two strategies are equalized when A’s probability is 2/3 and A’s payoffs from her two strategies are equalized when B’s probability is 1/3.

Player A chooses the mountain strategy (M) with probability r and the seaside strategy (S) with probability 1-r.

Player B chooses the mountain strategy with probability s and the seaside strategy with probability 1-s.
The key to finding the mixed strategy equilibrium (the set of probabilities that is an equilibrium for this game) is to choose A’s probabilities, r and 1-r, so that B is indifferent between M and S.

So, for Player B this looks like:

\[ 1r + 0(1-r) = 0r + 2(1-r) \]
\[ 1r = 2 - 2r \]
\[ 3r = 2 \]
\[ r = \frac{2}{3} \]

And, for Player A this looks like:

\[ 2s + 0(1-s) = 0s + 1(1-s) \]
\[ 2s = 1 - s \]
\[ 3s = 1 \]
\[ s = \frac{1}{3} \]

So, the mixed strategy equilibrium has Player A choosing M with probability 2/3 and S with probability 1/3, and Player B choosing M with probability 1/3 and S with probability 2/3.

Now, you try it for chicken. Here’s the payoff matrix:
Work through this example and try to find the mixed strategy equilibrium following the model presented above. Make sure that your answer seems sensible. That is, make sure that the probability with which they swerve or don’t makes sense.

What is the resulting probability that the really bad outcome (don’t, don’t) happens?

The Prisoners’ Dilemma

The most famous example of a game is the Prisoners’ dilemma. The story behind this game is that two people have been arrested in the process of committing a serious crime but can only be convicted of lesser offenses unless they confess and testify against each other. If neither confesses, they will both be convicted of lesser offenses and go to jail for one year each. If they both confess, they both plead guilty to the major crime and go to jail for six years each. If, however, one confesses and the other doesn’t, the confesser goes free and the other goes to jail for ten years. This game is diagrammed as

For each player, it is better to confess than to not confess given the other player’s action. Imagine that Player 1 chooses confess. Player 2 will go to jail for six years if he
confesses and for ten years if he doesn’t, so he is better off confessing. If Player 1 chooses Don’t, Player 2 will go free if he chooses to confess, but will spend one year in jail if he doesn’t, so he is better off confessing. No matter what Player 1 does, Player 2 is better off confessing.

Each player is better off confessing and the equilibrium in this game is for each player to confess. The problem is that this leads to the worst outcome (a total of twelve years in prison) rather than the best outcome (don’t, don’t, with a total of two years in prison).

The prisoners’ dilemma describes a lot of situations in which two participants who act in their own self-interest will play to an outcome or equilibrium that is the worst situation possible. This is in contrast to the usual economic model in which society’s best interest is achieved through individuals pursuing their own self-interest.

In terms of oligopolies and cartels, one can think of two firms in a market. They can each choose to produce a small quantity or a large quantity. If they both produce a small quantity, the price will be high and they will have large profits. If they both produce a large quantity, the price will be low and they will have small profits. However, if one firm is producing a small quantity, the other firm could take advantage of the relatively high price by producing a large quantity. The payoffs look like

<table>
<thead>
<tr>
<th></th>
<th>Small q</th>
<th>Large q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small q</td>
<td>+50</td>
<td>+60</td>
</tr>
<tr>
<td>Player 1</td>
<td>+50</td>
<td>+30</td>
</tr>
<tr>
<td>Large q</td>
<td>+30</td>
<td>+40</td>
</tr>
<tr>
<td>Player 2</td>
<td>+60</td>
<td>+40</td>
</tr>
</tbody>
</table>

For each firm, profits are larger from producing a large quantity than from producing a small quantity. The equilibrium is Large, Large even though the best outcome for the firms is Small, Small. This suggests that cartels would be difficult to sustain.

The outcome Large, Large is an example of a Nash equilibrium, a situation in which each player is doing the best that she can do given the other player’s action.

Of course, there doesn’t need to be a story behind the prisoners’ dilemma. It doesn’t even have to be symmetric:
For Player 1, 6 is better than 5 and 2 is better than 1, so Player 1 will choose Down.

For Player 2, 8 is better than 7 and 4 is better than 3, so Player 2 will choose Right.

The equilibrium is Down, Right.

The result is a total payoff of 6 (2+4) which is the worst of the four possible outcomes.

Now, some examples of Prisoners’ Dilemmas…

**Voting**
Society would probably be better off if all people took the time to become educated about issues and candidates and actually voted in elections. However, each person recognizes that their vote is extremely unlikely to decide an election and, as a result, probably won’t make any difference in the outcome. That being the case, each person would be better off if everyone else became educated and voted, but they themselves did not.

So, not becoming educated is the best strategy for each person, but if everyone does this society winds up at the worst possible outcome (lots of ignorant people and no one voting) rather than the best possible outcome (lots of educated people voting).

**Cleaning**
Two roommates share an apartment. They’re both a bit messy and each would be happier if they both put effort into cleaning, but each would be happiest if the other person did some cleaning while she herself did not. Not cleaning is a dominant strategy for each of the roommates and they wind up at the worst possible outcome, a really messy apartment.
The solution is for the roommates to get married (or at least have some sort of commitment ceremony with lots of family and friends in attendance) so that they know they’ll be playing against each other ‘til death do they part and then they may be able to sustain the good outcome.

State and Local Government Examples of Prisoners’ Dilemmas
Setting environmental regulations – should a state be strict or lax

Level of social services provision in a community – generous or not

Level of law enforcement provided in a community – high or low

Repeated Prisoners’ Dilemma
When a game is played over and over again by the same players, it is possible to maintain outcomes that are not equilibria in a one shot game (a game played only once).

Consider the Prisoners' Dilemma game in which the Nash equilibrium has both players defecting and arriving at the worst possible outcome rather than the best. If these players were to repeat the game some unknown number of times, the cooperative outcome could be sustained.
Consider a situation in which the same two players will play the above game an infinite number of times. Each player begins cooperating until the other defects and then responds by playing the defect strategy forever. After all, who could trust an opponent who has defected in the past?

Now, imagine that you’re going along cooperating and you do that forever. Your payoff stream will be:

8 8 8 8 8 8 8 8 8...

Now, imagine instead that you cheat and are then punished forever by your opponent defecting forever. Your payoff stream will be:

14 4 4 4 4 4 4 4 4...

If you like the first payment stream more than the second, you will continue to cooperate forever, despite the fact that you could get more, for one period, by defecting.

To be more clear about this, we can use discounting to calculate the present value of each stream and see if continued cooperation is likely to be sustainable. We will discount in the usual way using an interest rate of $i$. For example, the interest rate might be $i=0.05$, or 5%.

The present value of the continued cooperation payoff stream is:

$$PV = 8 + \frac{8}{1 + i} + \frac{8}{(1 + i)^2} + \frac{8}{(1 + i)^3} + \ldots = 8 + \frac{8}{i}$$
where the second term, $\frac{8}{i}$, is a result of an infinite sum equation that you’ve probably forgotten from your last calculus class.

The present value of the cheating payoff stream is:

$$PV = 14 + \frac{4}{1 + i} + \frac{4}{(1 + i)^2} + \frac{4}{(1 + i)^3} + \ldots = 14 + \frac{4}{i}$$

The interest rate that makes these equal is:

$$8 + \frac{8}{i} = 14 + \frac{4}{i}$$

$$\frac{4}{i} = 6$$

$$i = \frac{4}{6} = 0.667 = 66.7\%$$

So, if a player discounts the future at a rate of 66.7% per period, she will be indifferent between continued cooperation and cheating.

If a player discounts the future at a rate less than 66.7%, she will continue to cooperate.

For the most part, real interest rates are much lower than 66.7%, so we can expect that people discount the future at a lower rate than 66.7% and we would expect that a person playing this game would choose to continue cooperating.

However, if a person or a firm or a government became desperate or was facing imminent destruction, bankruptcy or overthrow, it might decide to cheat because it would have very little chance of existing in the long run.

**The Folk Theorem**
The so-called *Folk theorem* says that under some conditions, when the same people are playing a Prisoners’ dilemma type of game repeatedly, they can sustain an outcome preferable to the equilibrium.

For this reason, people who see each other and interact frequently often treat each other nicely.

One great example of this is in professional bicycle racing in which there is great consideration given for riders who crash. This is done because the same riders race against each other time and time again and failure to behave appropriately in one race can easily be punished in a later race.
Another great example of this was in World War I. Groups of soldiers who were stationed opposite one another for long periods of time would work themselves to good equilibria in which neither side would try to kill the other. They would intentionally shoot into the air or launch shells that would land nowhere near their “enemy’s” trenches. To thwart this, commanding generals would regularly rotate forces to frontline positions so that these better equilibria would not be achieved and the war would progress as usual.

When a Prisoners’ Dilemma is not infinitely repeated
If the game is played a known number of times, then the cooperative outcome can unravel from the last period back. In the last period firms will reach the equilibrium of the one-shot game (defect, defect). Knowing this, in the second to last period firms will also defect, realizing that there is no reason to cooperate since both firms will defect in the last period anyway.

Some Answers
The equilibrium for the truck stabbing game is:

I don’t recommend playing the game this way, but according to the game tree I should have thrown my keys into the swamp.

I don’t recommend playing the game this way because many criminals can’t solve a game tree through backward induction.
The matrix for the chicken game (which I also recommend not playing at all) is:

<table>
<thead>
<tr>
<th></th>
<th>1-s</th>
<th>Player B</th>
</tr>
</thead>
<tbody>
<tr>
<td>swerve</td>
<td>-10</td>
<td>+10</td>
</tr>
<tr>
<td>don't</td>
<td>-100</td>
<td>-100</td>
</tr>
</tbody>
</table>

To find the equilibrium probability \( r \), we do the following for Player B:

\[
\begin{align*}
\text{payoff to swerve} &= \text{payoff to don’t} \\
0r - 10(1-r) &= 10r - 100(1-r) \\
-10 + 10r &= 10r - 100 + 100r \\
90 &= 100r \\
r &= 0.9
\end{align*}
\]

So, Player A will swerve with probability 0.9.

To find the equilibrium probability \( s \), we do the following for Player A:

\[
\begin{align*}
\text{payoff to swerve} &= \text{payoff to don’t} \\
0s - 10(1-s) &= 10s - 100(1-s) \\
-10 + 10s &= 10s - 100 + 100s \\
90 &= 100s \\
s &= 0.9
\end{align*}
\]

So, Player B will swerve with probability 0.9.

The probability that both players will collide (don’t, don’t) is 0.1 x 0.1 = 0.01.

You really shouldn’t play chicken. It’s a bad game. However, if you have to play chicken, you should work hard to convince your opponent that you are totally insane and would rather die than lose.