Thus far, we have discussed two extreme market types: perfect competition and monopoly. Most real world markets lie somewhere between these two extremes.

**Pricing Under Homogeneous Oligopoly**

We’ll start with a basic model of *oligopoly*, or a market with few sellers.

In particular, we’ll imagine an oligopoly in which a small number of firms who are protected by some sort of barrier to entry are making a homogeneous good. Use of a homogeneous good allows us to talk about the price of that good without worrying about differences in quality or characteristics across producers.

The discussion here will differ somewhat from that in the book. We will focus on three different versions or outcomes of the Cournot model, in which firms compete by choosing the quantity that they will produce and then letting the market determine the market price through supply and demand.

A bit of notation will help.

The total market quantity will be given by $Q = q_1 + q_2 + \ldots + q_n$.

In regular English, this means that the market quantity, $Q$, will be the sum of the quantities produced by each of the $n$ firms in the industry, $q_1$, $q_2$ and so on up to $q_n$.

Price will be a function of $Q$. For example, it might be that price is given by:

$$P = 420 - 0.05Q$$

Each firm will have some cost structure based on its own quantity produced.

For example, firm 1 might have costs given by:

$$TC_1(q_1) = 200 + 2q_1^2$$

In this case, firm 1’s marginal cost function would be:

$$MC_1(q_1) = 4q_1.$$
We will typically identify two types of outcomes
1. The joint profit maximizing or cartel output level, which maximizes the sum of the firms' profits.
2. The Nash equilibrium, in which each firm acts in its own self-interest (to maximize its own profits) given the actions of the other firms.

The General Cournot model
In general, we will have an industry of n firms facing a demand function P(Q) in which each firm has costs TC(q_i) where q_i is the output of a particular firm and Q is the total firm output.

1. Solving for the joint profit maximizing output, choose individual firm outputs to

\[ \max Q \cdot P(Q) - TC(q_1) - \ldots - TC(q_n) \]

Taking the derivative with respect to each of the n firms’ quantities will yield n first order conditions with the general form

\[ P(Q) + Q \cdot \frac{\partial P}{\partial q_i} - \frac{\partial TC_i}{\partial q_i} = 0 \]

\[ P(Q) + Q \cdot \frac{\partial P}{\partial q_i} - MC_i(q_i) = 0 \]

Solving these simultaneously will yield the joint profit maximizing solution. If all firms are identical, it may simplify the problem at this point to assert that all firms will have the same output level instead of going through the trouble of solving simultaneously.

2. Solving for the Nash equilibrium, firms will choose their output level to

\[ \max q_i \cdot P(Q) - TC(q_i) \]

Taking the derivative with respect to each of the n firms’ quantities will yield n first order conditions with the general form

\[ P(Q) + q_i \cdot \frac{\partial P}{\partial q_i} - \frac{\partial TC_i}{\partial q_i} = 0 \]

\[ P(Q) + q_i \cdot \frac{\partial P}{\partial q_i} - MC_i(q_i) = 0 \]
Solving these simultaneously will yield the Nash equilibrium. If all firms are identical, it may simplify the problem at this point to assert that all firms will have the same output level instead of going through the trouble of solving simultaneously.

Joint profit maximizing solutions are difficult to maintain because, if all the other firms are producing the joint profit maximizing output, one firm can gain a huge profit by cheating or producing more than the joint profit maximizing output level.

The following example will apply the Cournot model of oligopoly competition, in which firms choose their quantities, to a duopoly, a market with two firms. We will first solve for the joint profit maximizing outcome, or the quantities that the firms would produce in order to maximize their combined profits. This is what they would do if they successfully formed a cartel. We will then solve for the Nash equilibrium, or the outcome when the firms do not cooperate, but rather compete in an attempt to maximize their own profit with no concern for the other firm’s profits. This will yield some reaction functions, or functions that specify each firm’s profit maximizing quantity as a function of the other firm’s quantity. The simultaneous solution of these reaction functions is the Nash equilibrium of the situation. Finally, we will consider what payoff can be had if the two firms are jointly maximizing profits and one decides to cheat on its cartel partner.

**EX:** An industry with two firms (called a duopoly) faces a market demand described by \( P=240-Q \) where \( Q \) is the sum of the individual firms' outputs. Each firm faces costs \( TC(q)=100+q^2 \).

1. Joint Profit Maximization
   To solve for the joint profit maximizing output, we want to choose each firm's output to maximize the sum of the two firms' profits:
   \[
   \max P(Q) \cdot Q - TC(q_1) - TC(q_2)
   \]
   \[
   \max (240 - Q) \cdot Q - (100 + q_1^2) - (100 + q_2^2)
   \]
   \[
   \max (240 - q_1 - q_2) \cdot (q_1 + q_2) - (100 + q_1^2) - (100 + q_2^2)
   \]
   where \( q_1 \) and \( q_2 \) are the output of the two firms and \( Q=q_1+q_2 \).

   Differentiating with respect to \( q_1 \) and \( q_2 \) yields
   \[
   (240 - q_1 - q_2) - (q_1 + q_2) - 2q_1 = 0
   \]
   \[
   (240 - q_1 - q_2) - (q_1 + q_2) - 2q_2 = 0
   \]
   or
\[ 240 - 4q_1 - 2q_2 = 0 \\
240 - 4q_2 - 2q_1 = 0 \]

(Note that \( q_1 = 120 - 2q_2 \) from the second equation.)

Solving these simultaneously yields \(^1\)

\[ 240 - 4(120 - 2q_2) - 2q_2 = 0 \\
240 - 480 + 8q_2 - 2q_2 = 0 \\
q_2 = 40 \]

Because these two firms are identical, we can conclude that \( q_1 = 40 \) as well.

The resulting profit is calculated by

\[ \pi_1 = P \cdot q_1 - TC(q_1) \]
\[ \pi_1 = (240 - q_1 - q_2) \cdot q_1 - (100 + q_1^2) \]
\[ \pi_1 = (240 - 40 - 40) \cdot 40 - (100 + 40^2) \]
\[ \pi_1 = 160 \cdot 40 - (100 + 1600) \]
\[ \pi_1 = 6400 - 1700 = 4700 \]

Because the two firms are identical, we also know that \( \pi_2 = 4700 \).

So the profit of each firm equals 4700.

2. Nash Equilibrium

Now, if the firms cannot cooperate to maximize their profits, each will choose its own output to maximize its own profit given the other's behavior. For instance, firm 1 will solve the problem:

\[ \max P(Q) \cdot q_1 - TC(q_1) \]
\[ \max (240 - Q) \cdot q_1 - (100 + q_1^2) \]
\[ \max (240 - q_1 - q_2) \cdot q_1 - (100 + q_1^2) \]

Differentiating with respect to \( q_1 \) yields

---

\(^1\) Actually, the easier way to do this, when the firms are identical, is to take the derivative and then assert symmetry, that is, because the firms are identical and acting simultaneously, \( q_1 = q_2 = q \).
Similarly, firm 2 will solve its maximization to yield

\[ 240 - 2q_2 - q_1 - 2q_2 = 0 \]
\[ 240 - 4q_2 - q_1 = 0 \]

(Note that \( q_1 = 240 - 4q_2 \).)

Solving these simultaneously yields \(^2\)

\[ 240 - 4(240 - 4q_2) - q_2 = 0 \]
\[ 240 - 960 + 16q_2 - q_2 = 0 \]
\[ 15q_2 = 720 \]
\[ q_2 = 48 \]

Again, because the two firms are identical, we can assert that \( q_1 = 48 \).

The resulting profit is calculated by

\[ \pi_1 = P \cdot q_1 - TC(q_1) \]
\[ \pi_1 = (240 - q_1 - q_2) \cdot q_1 - (100 + q_1^2) \]
\[ \pi_1 = (240 - 48 - 48) \cdot 48 - (100 + 48^2) \]
\[ \pi_1 = 144 \cdot 48 - (100 + 2304) \]
\[ \pi_1 = 6912 - 2404 = 4508 \]

So the profit of each firm equals 4508. This is less than their profits were when they were jointly maximizing their profits. For comparison, we have:

<table>
<thead>
<tr>
<th>Joint Profit Max</th>
<th>Nash Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q = 40 )</td>
<td>( q = 48 )</td>
</tr>
<tr>
<td>( \pi = 4700 )</td>
<td>( \pi = 4508 )</td>
</tr>
</tbody>
</table>

Incidentally, solving the first order conditions for the individual firm output levels yields

---

\(^2\) Again, you can assert symmetry after taking the derivative here.
These are called reaction functions, because they describe one firm's optimal action as a function of the other firm's action, and vice versa. The simultaneous solution of the reaction functions yields the Nash equilibrium.

EX: Returning to the above example, if one firm is producing at the joint profit maximizing level, what is the potential gain to the other firm is they deviate? To be more clear, imagine that there is a cartel and firm 1 is producing the prescribed quantity of 40 units, but that firm 2 turns out to be a dirty little cheater. What will happen?

3. Cheating
The joint profit maximizing level in the example is 40. If firm 1 is producing \( q_1 = 40 \) units, then the other firm faces the residual demand function

\[
P = 240 - q_1 - q_2 \\
P = 240 - 40 - q_2 \\
P = 200 - q_2
\]

and has costs

\[
TC(q_2) = 100 + q_2^2
\]

This yields the profit maximizing problem:
\[
\max q_2 \cdot (200 - q_2) - (100 + q_2^2) \\
\frac{\partial}{\partial q_2} = 200 - 2q_2 - 2q_2 = 0 \\
q_2 = 50
\]

Thus, \(Q=90\) and \(P=150\). In this case, the cheating firm's profit is:

\[
\pi_2 = 50 \cdot 150 - (100 + 50^2) = 4900.
\]

Thus, the cheating firm gains 200 over the profit they would have received at the joint profit maximizing outcome. Incidentally, firm 1 (the sucker) will have profits of

\[
\pi_1 = 40 \cdot 150 - (100 + 40^2) = 4300.
\]

So, while the cheater saw her profits rise by 200, the sucker saw her profits fall by 400.

**EX:**
Consider an oligopoly with two firms in which each firm's cost function is \(TC(q)=200 + q^2\) and demand is given by \(P(Q) = 840 - Q\).

1. Calculate the joint profit maximizing (JPM) output levels.

\[
\Pi_T = (840 - q_1 - q_2)( q_1+q_2) - 200 - q_1^2 - 200 - q_2^2
\]

Taking the derivative with respect to \(q_1\) yields:

\[
840 - 2q_1 - 2q_2 - 2q_1 = 0
\]

Now, after taking the derivative, because the firms are identical, we can assert that \(q_1 = q_2 = q\)

This gives us:
\[
840 - 6q = 0 \\
q = 140 \\
p=840 - 140 - 140 = 560 \\
\Pi_1 = 560*140 - 200 - 140^2 = 58600
\]

2. Calculate the Nash equilibrium output levels.

\[
\Pi_1 = (840 - q_1 - q_2) q_1 - 200 - q_1^2
\]

Taking the derivative with respect to \(q_1\) yields:

\[
840 - 2q_1 - q_2 - 2q_1 = 0
\]
Now, after taking the derivative, because the firms are identical, we can assert that 
$q_1 = q_2 = q$

This gives us:
$p = 840 - 168 - 168 = 504$
$\Pi_1 = 504*168 - 200 - 168^2 = 56248$

3. Calculate optimal cheating if one firm is producing the joint profit maximizing quantity. To make this fun, let’s imagine that firm 1 is producing the joint profit maximizing quantity of 140 and that firm 2 is going to cheat.

Firm 2 has the following profit function:
$\Pi_2 = (840 - 140 - q_2)*q_2 - 200 - q_2^2$

Taking the derivative with respect to $q_2$ gives:
$700 - 2q_2 - 2q_2 = 0$
$q_2 = 175$
$p = 840 - 140 - 175 = 525$
$\Pi_2 = (525 * 175) - 200 - 175^2 = 61050$
$\Pi_1 = (525 * 140) - 200 - 140^2 = 53700$

Under the joint profit maximizing outcome, both firms had profits of 58600.

The cheater’s (firm 2’s) profits increased by 2450.

The sucker’s (firm 1’s) profits decreased by 4900.

**Oligopoly**

Here’s a bunch of stuff about oligopolies that isn’t in the text, but which it is probably good to know, anyway.

Oligopoly is when there are a small number of firms in an industry. The concern in an oligopoly is that the firms will conspire to restrict output and raise prices and profits, and act known as cartelization, or collusion or, more politely, cooperative pricing.
Examples of industries that are oligopolistic are automobiles, breakfast cereals and beer brewing. In each of these, a small number of large firms dominate the market.

Characteristics of an Oligopoly
1. Consumers are price takers (many small consumers)
2. Firms produce homogeneous products (although this may be relaxed)
3. There is no entry
4. Firms do have market power
5. Each firm sets only its own price or output

Collusion or cartelization is a problem because it leads to smaller quantities and greater dead weight loss than would occur if the firms in the oligopoly were competing with each other more aggressively.

The fact that a cartel results in higher prices and profits means that the cartel transfers some wealth from consumers to the firms’ stockholders. Depending on your view of the world, this may or may not be a problem. However, cartels also generate dead weight loss because they increase the difference between marginal value and marginal cost.

Several factors facilitate cartelization
1. Ability to raise prices (this will depend on the price elasticity of demand)
2. Low expectations of punishment by the government
3. Small number of firms
4. A highly concentrated industry (Similar to there being few firms)
5. A homogeneous product (It is difficult to agree on a price if you’re selling different things)
6. Some forum for discussion such as trade associations
7. Ability to detect cheating and punish cheaters (organized crime is good at this)

David Besanko in *Economics of Strategy* offers these factors:

Market structure conditions affecting the sustainability of cooperative pricing
1. High Market Concentration Facilitates
2. Firm Asymmetries Harms
3. High Buyer Concentration Harms
4. Lumpy Orders Harms
5. Secret Price Terms Harms
6. Volatility of Demand and Costs Harms

as well as a number of practices which facilitate cooperative pricing
1. Price Leadership
2. Advance Announcement of Price Changes
3. Most Favored Customer Clauses
4. Uniform Delivered Pricing

**Measures of Concentration**
1. Four and Eight Firm Concentration Ratios
The four firm concentration ratio is the share of the market controlled by the four largest firms. The n firm concentration ratio is the share of the market controlled by the n largest firms.

For example, a market with five firms whose market shares are 30%, 25%, 20%, 15% and 10% would have a four firm concentration ratio of

\[30\% + 25\% + 20\% + 15\% = 90\% \text{ or } 0.90.\]

This would also be the four firm concentration ratio of a market whose firms had market shares of 87%, 1%, 1%, 1%, 0.5%, … 0.5%, even though the second industry is much more concentrated. One of the problems with concentration ratios is that they fail to account for what is happening with the N-n remaining firms.

2. Hirshmann-Herfindahl Index (HHI)
This is equal to the sum of the squared shares of all the firms in the market.

For example, a market with five firms whose market shares are 30%, 25%, 20%, 15% and 10% would have a HHI of

\[0.30^2 + 0.25^2 + 0.20^2 + 0.15^2 + 0.10^2 = 0.225\]

The more concentrated an industry is, the higher its HHI. The maximum HHI is 1, the minimum is 0.

An HHI gives you more information about an industry than a four firm concentration ratio does. Consider two markets, one served by five equally sized firms, the other served by four firms which each have a 20% share and twenty firms which have a 1% share. These would have the same four firm concentration ratios (0.80) but they would have different HHIs.

The interesting thing about the HHI is that a firm with an HHI equal to x can be expected to perform (in terms of price relative to marginal cost) similarly to an industry with 1/x equally sized firms.

An industry with an HHI of 0.25 could be expected to perform similarly to an industry with four equally sized firms.

For reasons I can't understand, the HHI is sometimes multiplied by 10,000. The Department of Justice will look into a merger if that merger will result in an industry's HHI increasing above about 1600 or 1700, which you might also call 0.16 or 0.17.

Concentration of Selected Manufacturing Industries from the 1992 Census of Manufactures
<table>
<thead>
<tr>
<th>Product Grouping</th>
<th>C4</th>
<th>C8</th>
<th>HHI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meat packing plant products</td>
<td>50</td>
<td>66</td>
<td>777</td>
</tr>
<tr>
<td>Cereal breakfast foods</td>
<td>85</td>
<td>98</td>
<td>2253</td>
</tr>
<tr>
<td>Distilled and blended liquors</td>
<td>62</td>
<td>82</td>
<td>1123</td>
</tr>
<tr>
<td>Cigarettes</td>
<td>93</td>
<td>Nr</td>
<td>Nr</td>
</tr>
<tr>
<td>Men’s and boy’s suits and coats</td>
<td>39</td>
<td>50</td>
<td>580</td>
</tr>
<tr>
<td>Wood kitchen cabinets</td>
<td>19</td>
<td>25</td>
<td>156</td>
</tr>
<tr>
<td>Folding paperboard boxes</td>
<td>23</td>
<td>35</td>
<td>237</td>
</tr>
<tr>
<td>Book publishing</td>
<td>23</td>
<td>38</td>
<td>251</td>
</tr>
<tr>
<td>Petroleum refining</td>
<td>30</td>
<td>49</td>
<td>414</td>
</tr>
<tr>
<td>Tires and inner tubes</td>
<td>70</td>
<td>91</td>
<td>1743</td>
</tr>
<tr>
<td>Blast furnaces and steel mills</td>
<td>37</td>
<td>58</td>
<td>551</td>
</tr>
<tr>
<td>Household refrigerators and freezers</td>
<td>82</td>
<td>98</td>
<td>1891</td>
</tr>
<tr>
<td>Motor vehicles and car bodies</td>
<td>84</td>
<td>91</td>
<td>2676</td>
</tr>
<tr>
<td>Games, toys and children’s vehicles</td>
<td>44</td>
<td>56</td>
<td>612</td>
</tr>
</tbody>
</table>

**Product Differentiation**

We will now turn our attention to the issue of product differentiation within a market. That is, we will look at goods that are similar enough in their function and use that they are considered to be part of the same market, but that are different enough so that any one good would not be considered a perfect substitute for another good by all consumers.

This brings up the issue of the definition of a market, which is a very tricky matter. The definition of a market, which is a critical step in anti-trust litigation, is a somewhat arbitrary matter. It is often unclear where to draw the lines that define a market.

For example, you might consider automobiles. Is it reasonable to talk about the market for domestic automobiles in the U.S. without also including foreign-made automobiles as well? Should light trucks and SUVs, which are also private modes of transportation, also be included? Should motorcycles, which provide similar transportation services, also be included?

At one point, a company (the Aluminum Company of America, or ALCOA) was accused of having established a monopoly in the market for, among other things, aluminum foil. While this was certainly true, their lawyers and economists argued that the wide variety of substitutes for aluminum foil, including wax paper, plastic wrap, butcher paper and the like were sufficient to keep ALCOA from raising the price of aluminum foil very much. This meant that, while they certainly had a monopoly in aluminum foil, they weren’t in a
position to significantly overcharge their customers as a result, so the existence of their monopoly didn’t hurt consumers and wasn’t a problem.

Anyhow, while we’ll assume away the problem of the differentiation of a market, you should recognize that it’s not such an easy issue.

The example on page 425 in the text can be thought of as a model of advertising, where that advertising is intended to create some differentiation, perhaps artificial differentiation, between one company’s product and another company’s product. If it helps to complete your mental picture, imagine that the two firms are both producing distilled water, but are spending money on advertising and packaging to try to differentiate their distilled water from that of their competitor(s).

The book presents the example with firms $i$ and $j$. I’ll do it for Firm1 and Firm 2.

Firm 1’s total costs are a function of the quantity of distilled water that it produces, $q_1$, and the amount it spends on differentiation or advertising, $z_1$:

$$ C_1 = C_1 (q_1, z_1) $$

Firm 1’s inverse demand function (or the price it can charge) is a function of its quantity produced, $q_1$, the price charged by Firm 2, $p_2$, the amount that Firm 1 spends on advertising, $z_1$, and the amount that Firm 2 spends on advertising, $z_2$:

$$ p_1 = p_1 (q_1, p_2, z_1, z_2) $$

with

$$ \frac{\partial p_1}{\partial q_1} \leq 0 \quad \frac{\partial p_1}{\partial p_2} \geq 0 \quad \frac{\partial p_1}{\partial z_1} \geq 0 \quad \frac{\partial p_1}{\partial z_2} \leq 0 $$

Firm 1’s profits are then:

$$ \pi_1 = p_1 (q_1, p_2, z_1, z_2) \cdot q_1 - C_1 (q_1, z_1) $$

and the first order conditions are:
\[
\frac{\partial \pi_1}{\partial q_1} = p_1 + \frac{\partial p_1}{\partial q_1} \cdot q_1 - \frac{\partial C_1}{\partial q_1} = 0
\]

\[
\frac{\partial \pi_1}{\partial q_1} = \text{MR} - \text{MC} = 0
\]

\[
\text{MR} = \text{MC}
\]

\[
\frac{\partial \pi_1}{\partial z_1} = \frac{\partial p_1}{\partial z_1} \cdot q_1 - \frac{\partial C_1}{\partial z_1} = 0
\]

The first is that marginal revenue should equal marginal cost. The second is that the marginal benefit of differentiation should be equal to the marginal cost. Unfortunately, this model doesn’t generate a lot of insight unless something is either known or assumed about the market and the effect of differentiation.

**Hotelling’s Linear Differentiation Model, the Linear City**

An important characteristic of a monopolistically competitive model is that different firms offer different products. One convenient way to think of how products differ in certain characteristics is to think about geographic location as a differing characteristic, and have firms choose their location.

This sort of model is called a Hotelling location model.

In this sort of a model, you imagine that consumers are geographically distributed and that sellers choose a location so as to attract as many customers as possible.

This is analogous to quantifiable characteristics of a product. For example, one important characteristic of ice cream is the butter fat content. If you know what percentage of butter fat other producers have in their products, you might choose a percentage that will let you take advantage of a gap in the market.

A linear version of this model has applications to political economy if you see voters as lying on a continuum from liberal to conservative and has fun implications for political conventions and the choices they make for presidential candidates. In particular, with a few assumptions, it can be shown that the median position dominates all others.

Example 14.3 goes through some math on a simple version of the linear model. The basic result is that if two firms are choosing positions along a line and each gets to serve the customers to which it is closer, they’ll wind up moving to the center. Both of them, right there at the center.
Entry in Oligopoly – Monopolistic Competition

When we started talking about oligopoly, one assumption was that there were barriers to entry. In fact, barriers to entry exist on a continuum and may be more or less severe. The important thing is that the degree to which extra-normal profits (positive economic profits) can be preserved over a long period of time depends on how strong the barriers to entry are. The stronger they are, the longer profits can be preserved. In addition, the greater the level of sustainable profits might be because very high profits will inspire entrepreneurs to find ways around barriers to entry. It is, after all, entry that eventually drives economic profits to zero.

The consideration of entry and many firms producing differentiated products gives up the type of market called monopolistic competition.

Now, in perfect competition, firms were price takers and entry drove profits to zero and drove prices down to minimum average cost so that products were produced as cheaply as possible:

However, firms in oligopolies are not price takers. They are free to choose the price that they will set, with the implication that each firm faces a downward sloping demand curve and a downward sloping marginal revenue curve, as in the monopoly model:
Now, if new firms enter the market, the effect will be to reduce the demand for each existing firm’s product.

Imagine the market for lunch in a downtown business area. There will be differentiation among burger places, pizza places, pho houses, fast Chinese places, sushi joints, pre-Columbian Mexican restaurants, Nepalese sherpa buffets, fried cheese stands, Philly steak sandwich carts, soup-on-a-stick vendors, strolling hot dog sellers, itinerant chocolatiers, falafel rollers, giro slicers, sandwich-making brew pubs, microsausageries, Thai noodle houses and what not. (I was sort of hungry when I wrote this.) When a new restaurant opens, it will take some customers from all of the pre-existing firms, and each of the pre-existing firms will perceive that demand for its particular product has fallen. This will happen despite the fact that the total demand for lunch in the area remains the same.

Anyhow, each individual firm will see demand for its goods dwindle as entry occurs until, in equilibrium, a typical firm’s economic profits are zero. However, as shown below, this will not happen at a quantity where average cost is minimized.
There are several problems with monopolistic competition
1. Because each firm faces downward sloping demand, in long run equilibrium average costs will not be minimized. As a result, output will not be produced at minimum cost.
2. Because each firm produces less than the cost minimizing quantity, there will be more firms than is efficient. To some extent, however, having too many firms is a tradeoff for having greater variety.