In this chapter, we will do some fun stuff with supply and demand and competitive analysis.

There is one potentially important caveat offered at the beginning of the chapter that I will echo here. The analysis offered here is partial equilibrium analysis, that is, it only looks at one market and doesn’t consider effects in other markets. A more complete analysis that looked explicitly at other markets and considered the interactions between markets might generate different results.

A large part of the focus of this chapter is on the gains from trade, which are a measure of the benefits to society of the exchanges that take place in a market. Gains from trade are divided into the gains to consumers, or consumers’ surplus, and the gains to producers, or producers’ surplus. These are shown for a market equilibrium in the following graph:

To see why gains from trade are maximized at $q^*$, it helps to think of the supply curve as a marginal cost curve, that is, the height of the supply curve gives the cost of making and selling one additional unit at each quantity, and to think of the demand curve as a marginal value curve, that is, the height of the demand curve gives the consumers’ marginal willingness to pay to get another unit of the good at each quantity. The supply curve is upward sloping because marginal cost is rising, at least in the short run. The demand curve is downward sloping because marginal value diminishes.
At quantities less than q*, the marginal value is greater than the marginal cost, so there are gains to be had by producing and selling additional units because consumers’ willingness to pay is greater than the marginal cost. For example, if I’m willing to pay $10 to get another unit of something and it costs you only $3 to make and sell it, there is the potential for an additional $10-$3=$7 in gains if an additional unit is made and sold. So, at quantities less than q*, it is reasonable to increase production.

At quantities greater than q*, the marginal value is less than the marginal cost, so there are gains to be had from reducing production because consumers’ willingness to pay is less than the marginal cost. For example, if I’m willing to pay $4 to get another unit of something and it costs you $12 to make and sell, making and selling that unit would result in a loss of $12-$4=$8. So, at quantities greater than q*, it is reasonable to reduce production.

If production is at some level other than q*, the gains from trade will be reduced. This reduction in gains from trade is called a dead weight loss or an efficiency loss.

For example, at a smaller quantity, q_s, there is the potential to produce additional units for which the marginal value is relatively high and the marginal cost is relatively low. Stopping at q_s results in the dead weight loss shown in the diagram below:

A graph showing the dead weight loss (or the foregone gains from trade) that result from too small a quantity being exchanged in a market.

Similarly, at too big a quantity, q_b, the last few units produced had a relatively high marginal cost and a relatively low marginal value and society would have been better off if they had not been produced at all. Production should be reduced at quantities greater than q*. 
A graph showing the deadweight loss (or losses in gains from trade) that result from too large a quantity of goods being exchanged in a market.

**Calculating This Stuff**

The book goes through a couple of calculations examples (Example 11.1) but I’ll offer a more complete set of examples here.

Imagine that you have supply and demand functions given by

\[ Q_D = 1200 - 2P \quad \text{and} \quad Q_S = 3P - 200 \]

We will solve for the equilibrium price and quantity and then calculate the dead weight loss resulting from quantities of 550 and 700.

First, to solve for equilibrium:

\[
\begin{align*}
Q_D &= Q_S \\
1200 - 2P &= 3P - 200 \\
1400 &= 5P \\
P^* &= 280, \quad Q^* = 640
\end{align*}
\]

Diagramming this, we have:
Further, we can calculate CS and PS:

\[
\text{CS} = \frac{1}{2} \cdot (600 - 280) \cdot (640 - 0) = \frac{1}{2} \cdot 320 \cdot 640 = 102,400
\]
\[
\text{PS} = \frac{1}{2} \cdot (280 - 67) \cdot (640 - 0) = \frac{1}{2} \cdot 213 \cdot 640 = 68,160
\]

Now, if for some reason the quantity produced is restricted to 550 units, there will be a resulting dead weight loss that can be calculated. To do this, we need to know the height of the demand and supply curves at the quantity \( Q=550 \). To calculate these values, we need to rewrite the supply and demand functions as marginal cost (MC) and marginal value (MV) functions:

\[
Q_D = 1200 - 2P \quad Q_S = 3P - 200
\]
\[
\text{MV} = P = 600 - Q/2 \quad \text{MC} = 200/3 + Q/3
\]

At \( Q=550 \) we have:

\[
\text{MV} = 600 - 550/2 \quad \text{MC} = 200/3 + (550/3)
\]
\[
\text{MV} = 600 - 275 \quad \text{MC} = 66.67 + 183.33
\]
\[
\text{MV} = 325 \quad \text{MC} = 250
\]

Now the calculation of the dead weight loss is simply a matter of finding the area of a triangle:
The dead weight loss (DWL) can be calculated as:

$$DWL = \frac{1}{2} \cdot (325 - 250) \cdot (640 - 550) = \frac{1}{2} \cdot 75 \cdot 90 = 3375$$

This is the value by which gains from trade would increase if the market were allowed to trade 640 units instead of 550.

You might be wondering if you could go one step further and describe how the dead weight loss or the extra gains from trade might be divided between consumers and producers. This could be done if you knew the market price when $Q=550$. If the market price at $Q=550$ is $P=280$, then figuring out the extra CS and extra PS from moving to $Q=640$ would be a fairly straightforward calculation of two triangles’ areas. However, the price at $Q=550$ is $P=325$, then moving to market equilibrium would represent a big increase in consumers’ surplus and, most likely, a decrease in producers’ surplus.

Now, if the quantity were 700, we would recreate the calculations from above:

At $Q=700$ we have:

$$\begin{align*}
MV &= 600 - 700/2 \\
MV &= 600 - 350 \\
MV &= 250
\end{align*}$$

$$\begin{align*}
MC &= 200/3 + (700/3) \\
MC &= 200/3 + (700/3) \\
MC &= 200/3 + (700/3)
\end{align*}$$
The dead weight loss (DWL) can be calculated as:

$$\text{DWL} = \frac{1}{2} \cdot (300 - 250) \cdot (700 - 640) = \frac{1}{2} \cdot 50 \cdot 60 = 1500$$

This is the value by which gains from trade would increase if the market quantity was cut from 700 back to 640. There would be increased gains from trade because the extra units, from unit 641 to unit 700, were all produced at a marginal cost that was greater than the associated marginal value. In other words, the resources that went into production of these extra sixty units would have been better used elsewhere.

Now, this sort of example can be re-done using the slightly more realistic constant elasticity demand functions. I say that these are more realistic because it is likely that you won’t know a linear equation for some real world market demand function, but you will probably have access to an estimate of the price elasticity of demand and will know a recent price and quantity and can come up with a combination of the two to get a demand function. I will duplicate the example from the book.

Imagine that you know that the price elasticity of demand for automobiles is $-1.2$ and you suspect, for whatever reason, that the price elasticity of supply is 1. This suggests that the demand and supply functions will be of the form:

$$Q_d = A \cdot P^{-1.2} \quad \quad Q_s = B \cdot P^1 = B \cdot P$$
Based on recent price and quantity information, you could come up with values to put in for A and B. The book uses A = 200 and B = 1.3. We can calculate the equilibrium as:

\[
Q_D = Q_S \\
200 \cdot P^{-1.2} = 1.3 \cdot P^{1.0} \\
\frac{200}{1.3} = \frac{P^{1.0}}{P^{1.2}} = P^{1.0-1.2} = P^{-0.2} = P^{2.2} \\
P^{2.2} = 153.846 \\
P^* = (153.846)^{\frac{1}{2.2}} = 9.866 \\
Q^* = 1.3 \cdot 9.866 = 12.825
\]

In a diagram, this looks like:

*A graph showing the equilibrium price and quantity from this example.*

Now, if the quantity exchanged in the market is restricted to 11 (million) units, there will be a dead weight loss. While this won’t exactly be a triangle, we’ll approximate the dead weight loss as a triangle. To do this, we need to calculate the MV and the MC at a quantity of 11:
The area of the triangle is given by:

\[
\text{DWL} = \frac{1}{2} \cdot (12.825 - 11) \cdot (11.21 - 8.46) = \frac{1}{2} \cdot 1.825 \cdot 2.75 = 2.51
\]

In a diagram, this looks like:

Finally, the example in the book talks about restricting the sale of new automobiles to control emissions of pollutants. This is a terrible idea for a pollution control policy and I need to tell you why. As the supply of new cars is restricted and their price rises from 9.866 to 11.21, demand for older, used cars will rise. Older cars, which tend to have more emissions that new cars, will be driven more and will be driven longer and this, if anything, will tend to make emissions problems worse rather than better.
Price Controls and Shortages

Figure 11.2 in the textbook is way more complicated than it needs to be. A more reasonable version looks like this:

Imagine that a market is subject to a price ceiling at $p_c$, which is below the equilibrium price. At the restricted price, $p_c$, the quantity demanded is $q_d$ and the quantity supplied is $q_s$, resulting in a shortage equal to $q_d - q_s$. The resulting dead weight loss is shown. The dead weight loss occurs because at $q_s$, additional units could be produced for which the MV is greater than the MC. So, producing and selling additional units would generate some additional gains from trade.

As an example, imagine that we have a linear system in which demand is given by $Q_D = 120 - P$ and supply is given by $Q_S = 2P - 30$. The equilibrium is $P=50$ and $Q=70$. Imagine that this market is subject to a price control of $P_c=40$. At this price, $q_d=80$ and $q_s=50$, with the resulting shortage of $80 - 50 = 30$. The calculation of the dead weight loss requires knowing the height of the demand curve or the marginal value at $q_s=50$. This is given by:

\[
\begin{align*}
Q_D &= 120 - P \\
P &= \text{MV} = 120 - Q \\
\text{MV(50)} &= 120 - 50 = 70
\end{align*}
\]

\[
\text{DWL} = \frac{1}{2} \cdot (70 - 50) \cdot (70 - 40) = 300
\]

This looks like:
A supply and demand graph showing the deadweight loss and the shortage resulting from a price ceiling.

**Non-price Rationing**

More should be said about price ceilings and shortages. When there is a shortage of some good, there must be some process to determine who gets the goods. Waiting in line is sometimes used as the rationing process, so that those who are willing to wait in line the longest get the stuff. In the above diagram the controlled price is $40 but the marginal willingness to pay at the quantity of 50 is $70. If people have a value of time of $10/hour, you might expect that the difference between the marginal value and the price will be made for by the time cost of waiting in line. $30 at $10/hour means an equilibrium waiting time of three hours. The resulting consumer surplus, including the cost of waiting in line, will be the small triangle at the top of the demand curve. The value of the time wasted waiting in line is the rectangle to the left of the dead weight loss triangle.

**Tax Incidence Analysis**

A fun question to ask is who bears the burden of a tax. Microeconomic theory says that it doesn’t matter on whom the tax is actually levied, the real burden of the tax depends on the relative elasticities of the consumers and suppliers in the market. When demand is relatively inelastic consumers will bear most of the burden of taxes. When supply is relatively inelastic suppliers will bear most of the burden of taxes. The standard diagram is shown below for cases of inelastic demand and of inelastic supply.
Two diagrams, side by side, demonstrating that the distribution of the burden of a tax depends on relative slopes of demand and supply curves. The first shows that when the demand curve is relatively steep, the consumer will bear most of the burden of the tax. The second shows that when the supply curve is relatively steep, the supplier will bear most of the burden of the tax.

The first diagram shows relatively inelastic (steep) demand. A tax imposed shifts the supply curve upward by the amount of the tax, leading to a reduction in the quantity exchanged and creation of a dead weight loss equal to the shaded area. After the tax, there is a higher price paid by the consumer (PPBC) and a lower price received by the supplier (PRBS), with the difference between these being equal to the per unit tax (PPBC – PRBS = tax). The two rectangles to the left of the dead weight loss triangle represent the total tax revenue. The top rectangle is the portion of the tax revenue taken from consumers in terms of reduced consumer surplus. The bottom rectangle is the portion of the tax revenue taken from producers, in terms of reduced producer surplus. The PPBC rises a lot as a result of the tax while the PRBS falls only slightly, so consumers, as a result of their inelastic demand, bear most of the burden of the tax. This is evidenced by the fact that their tax revenue rectangle (the top rectangle) is larger than the lower, suppliers’ rectangle.

The second diagram shows a relatively flat demand curve and a relatively steep supply curve, with the result being that suppliers bear most of the burden.

As an example, imagine that demand is given by \(Q_D = 120 - P\) and supply is given by \(Q_S = 2P - 30\). The equilibrium is \(P=50\) and \(Q=70\). For a tax of \$6, find the new equilibrium quantity, the new PPBC and PRBS, the total tax revenue raised, the reduction in consumers’ surplus, the reduction in producers’ surplus and the dead weight loss.

To find the new equilibrium, it is necessary to add the tax to the supply curve, but to do this you need to re-write the supply curve to get price in terms of quantity, add the tax, and then re-write the new supply curve in terms of quantity:

\[
Q_S = 2P - 30 \\
P = Q/2 + 15
\]
Tax: \[ P = \frac{Q}{2} + 15 + 6 \]
\[ P = \frac{Q}{2} + 21 \]
\[ Q_s = 2P - 42 \]

The new equilibrium is given by:

\[ 2P - 42 = 120 - P \]
\[ 3P = 162 \]
\[ P = PPBC = 54, \quad Q = 120 - 54 = 66 \]

\[ PRBS = PPBC - 6 = 54 - 6 = 48. \]

The diagram and resulting areas are:

A graph showing the numbers for the tax example given here.

Total tax revenue = \((54 - 48) \times 66 = 396\)

Reduction in CS = \((54 - 50) \cdot \frac{70 + 66}{2} = 272\)

Reduction in PS = \((50 - 48) \cdot \frac{70 + 66}{2} = 136\)

Dead Weight Loss = \(\frac{1}{2} \cdot (54 - 48) \cdot (70 - 66) = 12\)
The relationship between these is that the sum of the reduction in CS and the reduction in PS is equal to the sum of the tax revenue and the dead weight loss, which are both $408 in this example.

Put more clearly, the total damage to consumers and producers will be greater than the tax revenue collected, so taxes result in a welfare loss despite the fact that the money collected in tax revenues is merely transferred from one part of society (private individuals) to another part of society (the government).

The book goes through and establishes that how the tax burden is divided between consumers and suppliers depends on their elasticities. For example, if the price elasticity of demand is –1.2 and the price elasticity of supply is 0.6 the division of a $10 per unit tax to consumers and producers will be:

\[
\begin{align*}
\text{Consumers: } t \cdot \frac{\varepsilon_s}{\varepsilon_s - \varepsilon_d} &= $10 \cdot \frac{0.6}{0.6 + 1.2} = $3.33 \\
\text{Suppliers: } t \cdot \frac{\varepsilon_d}{\varepsilon_s - \varepsilon_d} &= $10 \cdot \frac{1.2}{0.6 + 1.2} = $6.67
\end{align*}
\]

So, the price paid by consumers, who are more flexible as evidenced by their elasticity of –1.2, will see the price that they pay rise by $3.33.

The price received by suppliers, who are less flexible as evidence by their elasticity of 0.6, will see the price that they receive fall by $6.67.

The textbook also gives an equation for dead weight loss as a function of the amount of the tax (dt) and the elasticities of supply and demand, \(\varepsilon_s\) and \(\varepsilon_d\), respectively. The equation is on page 324 and there is a related example, Example 11.2.

**International Trade and Restrictions**

Imagine that a small country that has not traded with the rest of the world in some good (wheat, perhaps) is considering opening up their market to international trade. The world wheat price is less than their domestic wheat price, so the price that consumers pay for wheat in the country will fall. Producers, of course, are opposed to opening trade to foreign producers and claim that they will all be driven out of business.

If the country is small, its entry into the world market won’t affect world prices, so world prices can be taken as given. The effect of the entry into world trade on the domestic wheat market is shown in the following diagrams:
Three graphs, side by side, showing the impact of opening trade on a domestic market. The first graph shows the increased consumer surplus for domestic consumers, the second shows the decreased producer surplus for domestic producers, and the third shows the difference between the two, or the domestic net gains.

As the price within the country falls from \( p^* \) to the world price \( p_w \), the total quantity purchased will rise from the domestic equilibrium \( q^* \) to \( q_1 \) and consumer surplus will increase.

As the price within the country falls, the quantity supplied by domestic suppliers will fall from \( q^* \) to \( q_2 \) and producer surplus will fall.

The net effect on domestic gains from trade will be the area shown in the third diagram, the difference between the consumer surplus gains and the producer surplus loss. It should be noted that the gains to consumers will outweigh the losses to producers, so the net benefits to opening up trade will be positive.

**Analysis of Tariffs**
Tariffs are often imposed on the import of foreign goods. In fact, this is how the U.S. government raised most of its tax revenue until the early 20th century.

The book presents this analysis graphically. The book’s analysis will be presented here with some additional explanation.

Let’s start with the original picture of international trade:
A simple supply and demand graph showing the net gains from the opening of a domestic market to international trade.

The shaded area represents the net gains from international trade and the fact that consumers can buy at the lower world price, $p_w$.

Now, imagine that a tariff equal to $t$ per unit is applied to the imported good. This will raise the effective price of the imported, world market goods from $p_w$ to $p_w+t$. This will make the net gains triangle smaller. The reduction in the net gains triangle can be divided into two parts. The rectangle is the tax revenue from the tariff, and is equal to the amount of the tariff multiplied by the quantity imported after the tariff is imposed. The remaining two triangles are the deadweight loss from the imposition of the tariff.

That is, the tax revenue is a transfer from domestic consumers to the government, so it isn’t a welfare loss. However, the reduction in gains from international trade that occurs as a result of the tariff is a welfare loss.
**Practice Problems and Stuff**

1. For a market with supply and demand given by $Q_S = 3P - 30$ and $Q_D = 90 - P$, find the following:
   a. equilibrium price and quantity
   b. CS and PS at the equilibrium
   c. shortage and DWL resulting from a price ceiling of $P=20$.
   d. surplus and DWL resulting from a price floor of $P=40$
   e. tax revenue and DWL from a tax of $12$ per unit.

2. For a market with supply and demand given by $Q_S = 1.2P$ and $Q_D = 20P^{1.5}$, find the following:
   a. equilibrium price and quantity
   b. shortage and approximate DWL resulting from a price ceiling of $P=3$.
   c. surplus and approximate DWL resulting from a price floor of $P=4$.
   d. tax revenue and approximate DWL from a tax of $10\%$. Note: this will require rewriting the supply curve so that it is $10\%$ higher than it is originally. To do this, solve the supply curve for $P$ and multiply by $1.10$, the re-solve for $Q$.
   e. for the tax revenue question in part d, is the tax burden distributed as theory predicts it should be? Explain.

3. In general, how the distribution of the burden of a tax depend on elasticity?

4. In general, how does the dead weight loss associated with a tax depend on elasticity?
5. If demand is given by $Q_s = 3P^{0.5}$ and demand is given by $Q_d = 12P^{-1.5}$ and a tax is imposed on the market, how will the burden of this tax be shared between suppliers and consumers?