This chapter discusses how consumers make consumption decisions given their preferences and budget constraints.

**A graphical introduction to the budget constraint and utility maximization**

A person will maximize their utility subject to their budget constraint. That is, they will do the best that they can given the amount of money they have to spend and the prices that they face.

If a person has income I and consumes goods x and y and the prices of these goods are $p_x$ and $p_y$, her budget constraint is written as:

$$ p_x x + p_y y \leq I $$

That is, the amount that she spends on x ($p_x$ multiplied by x) plus the amount that she spends on y ($p_y$ multiplied by y) must be less than or equal to her income, I.

For example, if a person has income of $120, the price of x is $10 per unit and the price of y is $5 per unit, then the budget constraint is written as:

$$ 10x + 5y \leq 120 $$

In terms of a graph, the budget constraint looks like:
Two graphs. The first shows a general budget line and the second shows the budget line for the situation where income is 120, the price of x is 10 and the price of y is 5.

The line represents the set of bundles that this person can afford if she spends all of her income on goods x and y. The slope of the budget constraint is $-\frac{p_x}{p_y}$.

Now, given that a person is constrained to choose a point on her budget line, she will try to get onto the highest indifference curve possible. This will occur at a point where the budget line is tangent to an indifference curve. In a diagram, this looks like:
where \((x^*, y^*)\) is the utility maximizing bundle of goods given the indicated preferences and budget constraint. Were the person at some other point on the budget line, she could make herself better off (that is, she could achieve a higher level of utility) by choosing a different combination of goods.

Now, here’s the thing. At the point \((x^*, y^*)\), the slope of the indifference curve is equal to the slope of the budget constraint, \(-\frac{p_x}{p_y}\), or the marginal rate of substitution. A point that maximizes a person’s utility must (with a few exceptions) be a point at which the two slopes are equal. So, at a utility maximizing point, it must be true that the slope of the budget line equals the slope of the indifference curve, which is also known as the marginal rate of substitution (MRS):

\[
\frac{-p_x}{p_y} = -\left.\frac{dy}{dx}\right|_{U=U} = MRS_{xy}
\]

**Example:**

Imagine that a person faces prices of \(p_x=\$20\) and \(p_y=\$30\) and has the utility function \(U(x,y) = x^2y\). The slope of her budget line would be

\[
\frac{-p_x}{p_y} = -\frac{20}{30} = -\frac{2}{3}
\]

The slope of her budget line would be

\[
MRS = -\frac{MU_x}{MU_y} = -\frac{\frac{\partial U}{\partial x}}{\frac{\partial U}{\partial y}} = -\frac{2xy}{x^2} = -\frac{2y}{x}
\]

giving us

\[
\frac{2}{3} = \frac{2y}{x} \quad x = 3y
\]

So, at a utility maximizing point, the quantity of \(x\) that she has will be equal to three times the quantity of \(y\) that she has. We don’t know exactly how much of each she’ll have without knowing her income or how much she has to spend, but we do know that possibilities include:

three units of \(x\) and one units of \(y\)
six units of x and two units of y
twelve units of x and four units of y
and so on.

**Some exceptions**
There are a few exceptions to the optimization rule stated above. That is, there are a few cases in which the utility maximizing point is not one where the slope of the indifference curve is equal to the slope of the budget line.

**Exception 1: Corner solutions**
It may be that the indifference curve is either always steeper or always flatter than the budget line. In this case, the utility maximizing bundle is entirely composed of one good or the other. The diagram for this looks like:

Two graphs showing indifference curves and budget lines in situations where the consumer chooses to consume either only good x or only good y.

**Exception 2: Perfect substitutes**
If two goods are perfect substitutes, then the indifference curve is simply a straight line and the analysis is similar to that of Exception 1. The result will either be that all of one good is consumed or that all of the other is consumed. Imagine two brands of gasoline that you consider to be identical. If the two brands have different prices, you would only consume the cheaper brand, other things being equal.
The one exception is when the slope of the indifference curve (which, remember, is a straight line) is the same as the slope of the budget line (also a straight line), in which case the price ratio is equal to the MRS and any combination of the two goods is utility maximizing. So if the two brands of gasoline had the same price, it really wouldn’t matter which you consumed, or if you consumed a combination of the two.

**Exception 3: Perfect complements**

If two goods are perfect complements, then the utility maximizing outcome is to consume them in the appropriate ratio, regardless of their relative prices. So, you will consume an equal number of left shoes and right shoes and, at any one time anyway, you will use four times as many tires as you have automobiles. This diagram looks like:

\[ Q_y \]
\[ \frac{I}{p_y} \]
\[ \frac{I}{p_x} \]
\[ y^* \]
\[ x^* \]
\[ Q_x \]
\[ U \]

* A graph showing a budget line and indifference curves for perfect complements.

**The Math Behind Utility Maximization**

The math behind all this is as follows.

The goal is to choose a bundle of goods so as to maximize utility subject to the budget constraint. This is written as:

\[
\text{max } U = U(x_1, x_2, \ldots, x_n)
\]

subject to
This can be rewritten as a Lagrangian (see Chapter 2)

\[ L = U(x_1, x_2, \ldots, x_n) + \lambda (I - p_1 x_1 + p_2 x_2 + \ldots + p_n x_n) \]

From above, the stuff in parentheses following the \( \lambda \) is equal to zero, so creating the Lagrangian is really just taking the utility function and adding zero to it. You might ask, “Why go through all that trouble?” Well, it will be worth it.

Now, take the partial derivative of \( L \) with respect to each of the \( x \) terms and with respect to \( \lambda \) to get a whole bunch of equations that you set equal to zero. It’s basically a complicated maximization problem:

\[
\begin{align*}
\frac{\partial L}{\partial x_1} &= \frac{\partial U}{\partial x_1} - \lambda p_1 = 0 & MU_1 = \lambda p_1 \\
\frac{\partial L}{\partial x_2} &= \frac{\partial U}{\partial x_2} - \lambda p_2 = 0 & MU_2 = \lambda p_2 \\
\frac{\partial L}{\partial x_n} &= \frac{\partial U}{\partial x_n} - \lambda p_n = 0 & MU_n = \lambda p_n \\
\frac{\partial L}{\partial \lambda} &= I - p_1 x_1 + p_2 x_2 + \ldots + p_n x_n = 0 & I = p_1 x_1 + p_2 x_2 + \ldots + p_n x_n
\end{align*}
\]

All of these equations can then be solved simultaneously to find the optimal bundle.

So what? Well, first consider the relative quantities of any two goods, cleverly named good \( i \) and good \( j \). We know that:

\[ MU_i = \lambda p_i \]

\[ MU_j = \lambda p_j \]

So it must also be the case that:

\[ \frac{MU_i}{MU_j} = \frac{\lambda p_i}{\lambda p_j} = \frac{p_i}{p_j} \]

or that the ratio of the marginal utilities, the negative of the MRS, is equal to the ratio of the prices, which is the negative of the slope of the budget line.
It also turns out that

$$\lambda = \frac{\text{MU}_1}{p_1} = \frac{\text{MU}_2}{p_2} = \ldots = \frac{\text{MU}_n}{p_n}$$

that is, $\lambda$, is equal to the additional bang for a buck spent on each good, and that this is equal across goods. If this were not true, if one good offered more marginal bang for an additional buck than did some other good, then a consumer could make herself better off by spending less on other goods and more on that good. In an optimal situation, this sort of move is not possible because she is already as well off as she can be.

In fact, in this sort of problem, $\lambda$ has the interpretation of being the marginal utility of income. It is the increase in the level of utility that would be achieved if income were to increase by one unit.

**Example:**

Imagine that the utility function is $U(x,y)=5xy^2$, $p_x=2$ and $p_y=8$ and $I=240$.

1. Set up the Lagrangian
2. Solve for the optimal bundle
3. Calculate the resulting level of utility
4. Graph out the relevant curves
5. Calculate the marginal utility of income at the optimum

$$L = U(x,y) + \lambda (I - p_x x - p_y y)$$

$$L = 5xy^2 + \lambda (240 - 2x - 8y)$$

$$\frac{\partial L}{\partial x} = 5y^2 - 2\lambda = 0$$

$$\frac{\partial L}{\partial x} = 10x - 8\lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = I - p_x x - p_y y = 0$$

From the first two first-order conditions (the first two derivatives) we get:

$$\frac{5y^2}{10x} = \frac{2\lambda}{8\lambda} \rightarrow \frac{y}{2x} = \frac{1}{4} \rightarrow x = 2y$$

and the budget constraint is:

$$240 = 2x + 8y$$
If we substitute \( x = 2y \) into the budget constraint we get:

\[
240 = 2(2y) + 8y \\
240 = 4y + 8y \\
240 = 12y \\
y = 20 \\
x = 2y = 2(20) = 40
\]

We can confirm that this satisfies the budget constraint:

\[
2(40) + 8(20) = 80 + 160 = 240.
\]

The resulting utility level is \( U(40, 20) = 5(40)(20)^2 = 80,000 \)

In a picture, this looks like:

![Graph showing the solution to the preceding utility maximization example.](A graph showing the solution to the preceding utility maximization example.)

Now, the marginal utility of income, \( \lambda \), is equal to:

\[
\lambda = \frac{\text{MU}_x}{p_x} = \frac{5y^2}{2} = \frac{5 \cdot 20^2}{2} = \frac{2000}{2} = 1000
\]

\[
\lambda = \frac{\text{MU}_y}{p_y} = \frac{10xy}{8} = \frac{10 \cdot 40 \cdot 20}{8} = \frac{8000}{8} = 1000
\]
In Example 4.1, the textbook goes through a more general example along these lines. This form of utility function is called a Cobb-Douglas utility function. The general form is

$$U(x, y) = x^\alpha y^\beta$$

You should look through this example.

In particular, you should go through the calculations that get you from

$$U(x, y) = x^\alpha y^\beta \text{ and } I = p_x x + p_y y$$

to the demand functions for x and y,

$$x^* = \frac{\alpha I}{(\alpha + \beta)p_x}$$

$$y^* = \frac{\beta I}{(\alpha + \beta)p_y}$$

or, as expressed in the book, when $\alpha + \beta = 1$,

$$x^* = \frac{\alpha I}{p_x}$$

$$y^* = \frac{\beta I}{p_y}$$

**Indirect Utility Functions**

So, the underlying belief is that people maximize their utility given their preferences and income and the prices they face. Another way of stating this is that the quantity of each good that a person consumes is a function of preferences, income and prices.
Now, because utility is a function of quantities consumed, and quantities consumed are functions of preferences, income and prices, then utility can be expressed as a function of preferences, income and prices, assuming that a person maximized their utility.

This sort of utility function, where utility is a function of preferences, income and prices is called an indirect utility function.

Put somewhat differently, the usual utility function is:

$$U(x_1, x_2, \ldots, x_n)$$

but, forgetting about preferences for a moment, the optimal quantity of each good consumed can be expressed as a function of prices and income:

$$x_1^* = x_1(p_1, p_2, \ldots, p_n, I)$$
$$x_2^* = x_2(p_1, p_2, \ldots, p_n, I)$$
$$x_n^* = x_n(p_1, p_2, \ldots, p_n, I)$$

So, maximum utility can be expressed as

$$U(x_1^*, x_2^*, \ldots, x_n^*) = V(p_1, p_2, \ldots, p_n, I)$$

In terms of the utility function given above, $$U(x, y) = 5xy^2$$, the demand functions for x and y are:

$$x^* = \frac{\alpha I}{(\alpha + \beta)p_x} = \frac{1}{(1 + 2)p_x} = \frac{1}{3p_x}$$
$$y^* = \frac{\beta I}{(\alpha + \beta)p_y} = \frac{21}{(1 + 2)p_y} = \frac{21}{3p_x}$$

So the indirect utility function is:

$$V(p_x, p_y, I) = U(x^*(p_x, p_y, I), y^*(p_x, p_y, I)) = 5 \left( \frac{21}{3p_y} \right)^2 = \frac{20}{27} \frac{I^3}{p_x p_y^2}$$
We can confirm that for $p_x=2$ and $p_y=8$ and $I=240$ the resulting utility level is 80,000:

$$x^* = \frac{\alpha I}{(\alpha + \beta)p_x} = \frac{1}{3p_x}$$

$$y^* = \frac{\beta I}{(\alpha + \beta)p_y} = \frac{2I}{3p_x}$$

$$V(p_x, p_y, I) = \frac{20}{27} \cdot \frac{I^3}{p_xp_y^2} = \frac{20}{27} \cdot \frac{240^3}{2 \cdot 8^2} = 80,000.$$ 

**The Lump Sum Principle**

OK, you’ve suffered through enough theory with no obvious policy implications, so here’s something that can be applied to the real world. The idea is that if a tax is going to be imposed on a person, it is better to impose it as a lump sum tax (you pay $X$, regardless of your behavior) rather than taxing one thing or another. To state this more specifically, the same amount of tax revenue can be raised with less of a decrease in utility with a lump sum tax than with a tax on one good or another. This statement can be established based on only the simplest principles of consumer preferences and utility maximization.

Again, this statement can be established based on only the simplest principles of consumer preferences and utility maximization. You don’t need to know anything more.

Here’s the story in pictures:

A happy consumer is minding her own business, with income level $I$, facing prices $p_x$ and $p_y$, and achieving utility level $U_3$ as a result.
Now, for reasons we don’t need to go into here (refer to Chapter 20), the Government decides that it needs some tax revenue. As such, it must impose a tax.

To start with, imagine that they tax good \( x \). The choice of which good to tax is completely arbitrary, but you might imagine that some sort of excuse is given for choosing \( x \) over \( y \). As a result, the price of good \( x \) rises to \( p_x + t \), or the price plus the tax.\(^1\)

Anyhow, with a tax on \( x \), the picture changes to:

\(^1\) I can’t believe I’m putting a footnote in lecture notes, but it is worth saying that a tax equal to \( t \) won’t necessarily raise the price by \( t \). In general it will raise the price by less than \( t \), but under some conditions the price might rise by the full amount of the tax.
So, now she’s at a lower utility level and the Government is collecting some amount of tax revenue.

What if the government collected these taxes through a lump sum tax rather than a tax on \( x \). That is, what if the tax revenue stayed the same, but the prices of the goods stayed the same?

The new budget line would have the same slope as the original budget line, but would pass through the optimal point that the consumer achieved with the tax on \( x \).

The budget lines would look like:

\[
\begin{align*}
&\text{A graph showing the impact on the consumer's budget line of a tax on } x \text{ and the impact of a lump sum tax.}
\end{align*}
\]

Now, here’s the point. With the lump sum tax instead of the tax on \( x \), this consumer can achieve a higher level of utility without tax revenues changing. In the textbook, Figure 4.5 shows this new utility level as \( U_2 \).
A diagram showing that a consumer can achieve greater utility under a lump sum tax than under a revenue-equivalent tax on good x.

The implication is that lump sum taxes will be more efficient than will any other sort of tax, including taxes on goods, sales, income, property or labor. That is, the same amount of revenue can be raised with less of a decrease in consumer utility with a lump sum tax than with any other kind of tax.

Now, let’s try this with a particular utility function and income levels. Imagine that a consumer’s utility function is \( U(x, y) = x^{0.5} y^{0.5} \), her income is 480 and the prices of x and y are \( p_x = 1 \) and \( p_y = 1 \). With no tax in place, the maximization problem is:

\[
L = x^{0.5} y^{0.5} + \lambda (480 - x - y)
\]

The first derivatives are:

\[
\frac{\partial L}{\partial x} = \frac{y^{0.5}}{2x^{0.5}} - \lambda = 0 \quad \rightarrow \quad \frac{y^{0.5}}{2x^{0.5}} = \lambda
\]

\[
\frac{\partial L}{\partial y} = \frac{x^{0.5}}{2y^{0.5}} - \lambda = 0 \quad \rightarrow \quad \frac{x^{0.5}}{2y^{0.5}} = \lambda
\]

\[
\frac{\partial L}{\partial \lambda} = 480 - x - y = 0 \quad \rightarrow \quad 480 = x + y
\]

Taking the ratio of the first to first derivatives gives us:
This is combined with the budget constraint \( x + y = 480 \) with the result being that \( x=240 \) and \( y=240 \) and the utility level is \( U=240^{0.5}240^{0.5}=240 \).

Now, imagine that a tax of $1 is put on \( x \), raising the price of \( x \) to \( p_x=2 \). The new result will be \( x=120 \) and \( y=240 \) with a resulting utility level of \( U=120^{0.5}240^{0.5}=169.7 \). That tax revenue will be $1 per unit of \( x \), collected on 120 units of \( x \) for total tax revenue of $120.

Now, imagine that instead of a tax on \( x \), there was just a lump sum tax of $120 imposed, bringing this person’s disposable income from $480 down to $360. With the original prices of \( p_x=1 \) and \( p_y=1 \), we get a utility maximizing bundle (you should calculate this yourself and make sure you can do it) of \( x=180 \) and \( y=180 \) and a utility level of \( U=180^{0.5}180^{0.5}=180 \), instead of the utility level of 169.7 with the tax on \( x \).

Now, this can be expressed in terms of the indirect utility function. That is, when utility is expressed as a function of income and prices instead of as a function of the quantities of goods consumed, you can repeat the analysis.

For the utility function given above, \( U(x, y) = x^{0.5}y^{0.5} \), the indirect utility function can be found by calculating the demand functions for \( x \) and then inserting these into the utility function.

The demand functions for \( x \) and \( y \), in terms of \( p_x \) and \( p_y \) and income can be found by maximizing utility subject to the budget constraint:

\[
L = x^{0.5}y^{0.5} + \lambda \left( I - p_x x - p_y y \right)
\]

The first derivatives are:
\[
\frac{\partial L}{\partial x} = \frac{y^{0.5}}{2x^{0.5}} - \lambda p_x = 0 \rightarrow \frac{y^{0.5}}{2x^{0.5}} = \lambda p_x \\
\frac{\partial L}{\partial x} = \frac{x^{0.5}}{2y^{0.5}} - \lambda p_y = 0 \rightarrow \frac{x^{0.5}}{2y^{0.5}} = \lambda p_y \\
\frac{\partial L}{\partial \lambda} = I - x - y = 0 \rightarrow I = p_x x + p_y y
\]

The first two of these can be combined to give:

\[
y = \frac{p_x x}{p_y} \quad \text{and} \quad x = \frac{p_y y}{p_x}
\]

These can be combined with the third equation to give:

\[
I = p_x x + p_y \frac{p_x x}{p_y} = 2 p_x x \rightarrow x^* = \frac{I}{2 p_x} \\
I = p_y y + p_x \frac{p_y y}{p_x} = 2 p_y y \rightarrow y^* = \frac{I}{2 p_y}
\]

And this can be combined with the utility function itself to give:

\[
V(p_x, p_y, I) = \left( \frac{I}{2 p_x} \right)^{0.5} \left( \frac{I}{2 p_y} \right)^{0.5} = \frac{I}{2 p_x^{0.5} p_y^{0.5}}
\]

Now, at long last, we return to this nasty issue of lump sum taxes.

The initial situation was \(I=480, p_x=1, p_y=1\), for a utility level of:

\[
V(1,1,480) = \frac{480}{2 \cdot 1 \cdot 1} = 240.
\]

When the price of \(x\) rose to $2, the level of utility was:

\[
V(2,1,480) = \frac{480}{2 \cdot 2^{0.5} \cdot 1} = 169.7.
\]

Finally, when disposable income fell to $360 due to a lump sum tax of $120, the level of utility was:

\[
V(1,1,360) = \frac{360}{2 \cdot 1 \cdot 1} = 180.
\]
In a diagram, this all looks like:

A graph showing the numerical results of an example demonstrating that a consumer can achieve greater utility under a lump sum tax than under a revenue equivalent tax on good x.

**Expenditure Minimization**

So, there are two ways to look at a consumer’s optimal decision, but it’s all the same problem.

The first way is to imagine that the consumer has some budget constraint and tries to maximize her utility level given that budget constraint.

The second way is to imagine that the consumer will achieve some level of utility and tries to minimize the cost of doing this.

The answer to both of these problems is a point where the budget constraint is just tangent to an indifference curve.

In terms of a Lagrangian, these two problems are written as:

\[
L = U(x_1, x_2, \ldots, x_n) + \lambda (I - p_1 x_1 + p_2 x_2 + \ldots + p_n x_n)
\]

and

\[
L = (p_1 x_1 + p_2 x_2 + \ldots + p_n x_n) + \lambda \left( \bar{U} - U(x_1, x_2, \ldots, x_n) \right)
\]
The expenditure function is a function giving the minimum expenditure needed to achieve some level of utility, \( U \), given the prices for goods:

\[
E(p_1, p_2, \ldots, p_n, \overline{U}).
\]

Now, let’s calculate an expenditure function for the utility function \( U = U(x, y) = x^\alpha y^\beta \). The Lagrangian is:

\[
L = p_x x + p_y y + \lambda (\overline{U} - x^\alpha y^\beta)
\]

Taking first derivatives gives:

\[
\frac{\partial L}{\partial x} = p_x - \lambda \alpha x^{\alpha-1} y^\beta = 0 \rightarrow p_x = \lambda \alpha x^{\alpha-1} y^\beta
\]

\[
\frac{\partial L}{\partial y} = p_y - \lambda \beta x^\alpha y^{\beta-1} = 0 \rightarrow p_y = \lambda \beta x^\alpha y^{\beta-1}
\]

\[
\frac{\partial L}{\partial \lambda} = \overline{U} - x^\alpha y^\beta = 0 \rightarrow \overline{U} = x^\alpha y^\beta
\]

Now, taking the ratio of the first two of these gives:

\[
x = \frac{\alpha p_y y}{\beta p_x}
\]

\[
\frac{p_x}{p_y} = \frac{\alpha y}{\beta x} \rightarrow y = \frac{\beta p_x x}{\alpha p_y}
\]

Combining these with the other derivative gives:

\[
\overline{U} = x^\alpha y^\beta
\]

\[
U = x^\alpha \left( \frac{\beta p_x x}{\alpha p_y} \right)^\beta
\]

\[
\overline{U} = x^\alpha y^\beta
\]

\[
U = \left( \frac{\alpha p_y y}{\beta p_x} \right)^\alpha y^\beta
\]

\[
\overline{U} = x^{\alpha+\beta} \left( \frac{\beta p_x}{\alpha p_y} \right)^\beta
\]

\[
U = y^{\alpha+\beta} \left( \frac{\alpha p_y}{\beta p_x} \right)^\alpha
\]

\[
x^* = \left[ \frac{\beta p_x}{\alpha p_y} \right]^\beta \frac{1}{\alpha+\beta}
\]

\[
y^* = \left[ \frac{\alpha p_y}{\beta p_x} \right]^\alpha \frac{1}{\alpha+\beta}
\]
These functions are terribly ugly unless, as with Example 4.4 in the textbook, you assume that $\alpha = \beta = 0.5$, then they become

$$
x^* = \overline{U} \left( \frac{p_y}{p_x} \right)^{0.5} \quad y^* = \overline{U} \left( \frac{p_x}{p_y} \right)^{0.5}
$$

Now, the expenditure function will be:

$$
E(p_x, p_y, \overline{U}) = p_x x^* + p_y y^*
$$

$$
E(p_x, p_y, \overline{U}) = p_x \overline{U} \left( \frac{p_y}{p_x} \right)^{0.5} + p_y \overline{U} \left( \frac{p_x}{p_y} \right)^{0.5}
$$

$$
E(p_x, p_y, \overline{U}) = \overline{U} p_x^{0.5} p_y^{0.5} + \overline{U} p_x^{0.5} p_y^{0.5}
$$

$$
E(p_x, p_y, \overline{U}) = 2 \overline{U} p_x^{0.5} p_y^{0.5}
$$

**Properties of expenditure function**

1. **Homogeneity**
   
   Homogeneity means that if one of the components of a function increases by a certain percentage, the value of the function will increase by that percentage raised to some power.

   For example, if $y = f(x)$ is homogeneous of degree 6, then the following relationship will be true:

   $$
f(ax) = a^6 f(x)
$$

   Now, expenditure functions are homogeneous of degree 1 with respect to changes in prices. This is a fancy way of saying that if prices rise by 10%, expenditures will rise by 10%, holding utility constant. In an equation, this is:

   $$
   E(1.10 p_x, 1.10 p_y, U) = 1.10 \times E(p_x, p_y, U)
   $$

   If prices double, expenditures will double.

2. **Expenditure functions are increasing in prices.**

   If prices go up, expenditures will rise, holding utility constant.
3. **Expenditure functions are concave in prices.**

As the price of one good rises, holding other things constant, expenditures will rise at a slower rate than the rate at which prices rise because, in some sense, people will substitute toward the other good whose price hasn’t risen.
Exercises

1. Do the following for the utility function U(x,y) = xy
   A. Solve for the optimal bundle if p_x=1, p_y=1, I=120.
   B. Solve for the optimal bundle if p_x=1, p_y=1, I=240.
   C. Solve for the optimal bundle if p_x=2, p_y=1, I=120.
   D. Solve for the optimal bundle if p_x=1, p_y=2, I=120.
   E. Solve for the optimal bundle if p_x=2, p_y=2, I=240.
   F. Solve for the indirect utility function V(p_x, p_y, I).
   G. Solve for the expenditure function E(p_x, p_y, U).

2. Do the following for the utility function U(x,y) = x^{0.5}y^{0.5}
   A. Solve for the optimal bundle if p_x=1, p_y=1, I=120.
   B. Solve for the optimal bundle if p_x=1, p_y=1, I=240.
   C. Solve for the optimal bundle if p_x=2, p_y=1, I=120.
   D. Solve for the optimal bundle if p_x=1, p_y=2, I=120.
   E. Solve for the optimal bundle if p_x=2, p_y=2, I=240.
   F. Solve for the indirect utility function V(p_x, p_y, I).
   G. Solve for the expenditure function E(p_x, p_y, U).

3. Do the following for the utility function U(x,y) = xy^{0.5}
   A. Solve for the optimal bundle if p_x=1, p_y=1, I=120.
   B. Solve for the optimal bundle if p_x=1, p_y=1, I=240.
   C. Solve for the optimal bundle if p_x=2, p_y=1, I=120.
   D. Solve for the optimal bundle if p_x=1, p_y=2, I=120.
   E. Solve for the optimal bundle if p_x=2, p_y=2, I=240.
   F. Solve for the indirect utility function V(p_x, p_y, I).
   G. Solve for the expenditure function E(p_x, p_y, U).

4. Do the following for the utility function U(x,y) = x^2y
   A. Solve for the optimal bundle if p_x=1, p_y=1, I=120.
   B. Solve for the optimal bundle if p_x=1, p_y=1, I=240.
   C. Solve for the optimal bundle if p_x=2, p_y=1, I=120.
   D. Solve for the optimal bundle if p_x=1, p_y=2, I=120.
   E. Solve for the optimal bundle if p_x=2, p_y=2, I=240.
   F. Solve for the indirect utility function V(p_x, p_y, I).
   G. Solve for the expenditure function E(p_x, p_y, U).

5. Do the following for the utility function U(x,y) = x^2y^2
   A. Solve for the optimal bundle if p_x=1, p_y=1, I=120.
   B. Solve for the optimal bundle if p_x=1, p_y=1, I=240.
   C. Solve for the optimal bundle if p_x=2, p_y=1, I=120.
   D. Solve for the optimal bundle if p_x=1, p_y=2, I=120.
   E. Solve for the optimal bundle if p_x=2, p_y=2, I=240.
   F. Solve for the indirect utility function V(p_x, p_y, I).
   G. Solve for the expenditure function E(p_x, p_y, U).